Highly Accurate Quasi-Static Modeling of Microstrip Lines Over Lossy Substrates

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Abstract—**A highly accurate quasi-static model of a microstrip over a semiconductor layer has been developed. The model agrees with full-wave calculations in all three modes of propagation (skin-effect, slow-wave, and dielectric quasi-TEM), for both the attenuation constant α and the propagation constant β over a very wide range of dimension, substrate conductivity, and frequency. To achieve this level of agreement, a nonuniform cross-section, transverse resonance technique has been applied to find the series impedance per unit length of the microstrip transmission line.**

I. Introduction

For the last twenty-five years there has been a great deal of interest in modeling microstrip transmission lines on semiconducting substrates. Interconnects fabricated on multi-layered semiconductor substrates (such as silicon dioxide on silicon) produce behavior that is more difficult to predict than that of lines made on lossless substrates [1], [2]. In 1971, Hasegawa *et al.* [3] experimentally verified this behavior for a microstrip on an SiO_2–Si substrate. The purpose of this letter is to show that quasi-static analysis can accurately predict the behavior of such transmission lines, with excellent agreement between full-wave and static model over a very wide range of dimension, substrate conductivity, and frequency.

To evaluate the impact of a semiconductor layer of conductivity σ on the transmission line changes in both electric and magnetic fields must be determined. For a microstrip-like geometry (Fig. 1) the changes in the electric field are relatively straightforward. If the frequency of the applied signal is below the dielectric relaxation frequency of the semiconductor $\sigma/\varepsilon_{\text{semi}}$, the electric fields behave as if the semiconductor were a metallic sheet. Conversely, if the frequency is increased or conductivity decreased until $\omega > \sigma/\varepsilon_{\text{semi}}$, the electric fields behave as if the semiconductor were a lossy dielectric layer. In the crossover region where $\omega \sim \sigma/\varepsilon_{\text{semi}}$, the impact of the semiconductor conductivity on propagation loss can be very large [4].

The proper value of series inductance for the transmission line must also be determined. When the thickness of the semiconducting substrate becomes greater than the skin depth, the so-called "skin-effect" mode of propagation is encountered [3]. Several previous papers have recognized that this leads to a reduction in the effective separation between the signal

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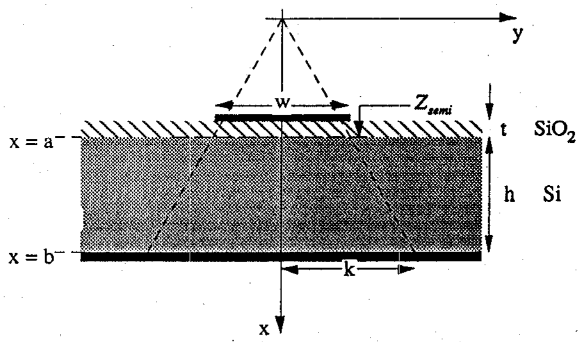

Fig. 1. Cross section of a microstrip over a semiconducting substrate. k represents the effective spreading distance of the fields between the strip and the ground plane; best agreement between this model (5) and conventional microstrip calculations is achieved for $k = 3h + w/2$.

and the ground plane, as well as inducing significant loss due to series resistance [3], [5]. In contrast, if the frequency or conductivity is low enough that the skin depth is larger than the thickness of the semiconductor, the magnetic fields (and thus inductance L) will be determined primarily by the separation of the microstrip and the true ground plane.

II. Quasi-Static Model

The use of quasi-static models for transmission lines is well established, and has great utility when computational efficiency is required [6]. In addition, for cases where it is essential to include conductor losses, full-wave analysis can become arduous. Many quasi-static models have been proposed for microstrips over semiconducting layers that adequately describe the impact of finite conductivity on the shunt admittance per unit length Y of the transmission line [7]. The equivalent circuit used consists of a capacitor C_{insu}, representing the top dielectric layer, in series with a parallel capacitance C_{semi} and conductance G_{semi}, representing the semiconducting layer. For simplicity, we assume that the top dielectric layer thickness t is much less than the microstrip width w, so

$$C_{\text{insu}} = \frac{\varepsilon_{\text{insu}}}{t} w, \tag{1}$$

where $\varepsilon_{\text{insu}}$ is the dielectric constant of the top insulating layer. For the semiconducting layer, the shunt conductance G_{semi} scales identically with its capacitance. Here, we use Wheeler's equations [8] to find the quasi-static capacitance due to the semiconductor portion of the interconnect, C_{semi}, and then the conductance is

$$G_{\text{semi}} = \frac{\sigma}{\varepsilon_{\text{semi}}} C_{\text{semi}}, \tag{2}$$

where e_{semi} is the dielectric constant of the semiconductor. The use of Wheeler's equations efficiently accounts for thickness variations of the semiconductor layer h with respect to the microstrip width. Thus, the total admittance per unit length for the interconnect is given by

$$Y = \frac{j\omega C_{\text{insu}} G_{\text{semi}} - \omega^2 C_{\text{semi}} C_{\text{insu}}}{G_{\text{semi}} + j\omega(C_{\text{semi}} + C_{\text{insu}})}. \tag{3}$$

The semiconductor layer can also significantly affect the series impedance per unit length Z of the microstrip. For the high-frequency and/or high-conductivity case, this effect has not been adequately treated in previous quasi-static models. Here, we use the transverse resonance technique to find the surface impedance of the ground plane as seen through the semiconductor layer, similar to that previously used by [5]. Previous work assumed that the equivalent transverse transmission line is of uniform cross section with a short circuit boundary condition representing the perfect ground plane. A uniform cross section approximation, however, is invalid for microstrip except for very wide strips (i.e., $w >> h$). If the effective cross section is assumed to vary linearly with depth x (Fig. 1), approximating the spreading of the fields between the microstrip and the ground plane, the input impedance of this nonuniform transmission line is the desired surface impedance, and is given by (4) below, where $\boldsymbol{H}_n^{(1)}$ and $\boldsymbol{H}_n^{(2)}$ are Hankel functions of the first and second kind, $\beta_s = \sqrt{j\omega\mu_0(j\omega\varepsilon_{\text{semi}} + \sigma)}$, $a = (hw)/(2k - w)$, and $b = a + h$. The distance k is a measure of how much the fields spread before reaching the ground plane. The total impedance per unit length for the microstrip is then

$$Z = Z_i \frac{Z_{\text{semi}} + Z_i \tanh(\gamma_i t)}{Z_i + Z_{\text{semi}} \tanh(\gamma_i t)}, \tag{5}$$

where $Z_i = \left(\sqrt{\mu_0/\varepsilon_{\text{insu}}}\right)/w$ and $\gamma_i = j\omega\sqrt{\mu_0\varepsilon_{\text{insu}}}$. In the limit of zero conductivity in the semiconductor, the impedance calculated using (5) should reduce to the inductance of a simple microstrip line. For $k = 3h + w/2$, the inductance calculated using (5) matches that obtained from Wheeler's equations [8] very closely (within 3%) over a wide range of h/w (height–width ratio range of at least 10^{-2} to 10^2).

III. Results

The complex propagation constants of two microstrip line structures have been calculated using both full-wave and the new quasi-static model. The spectral domain approach is used for the full-wave calculations [9]. The full-wave results shown here are essentially identical to the results of Mesa *et al.* [10].

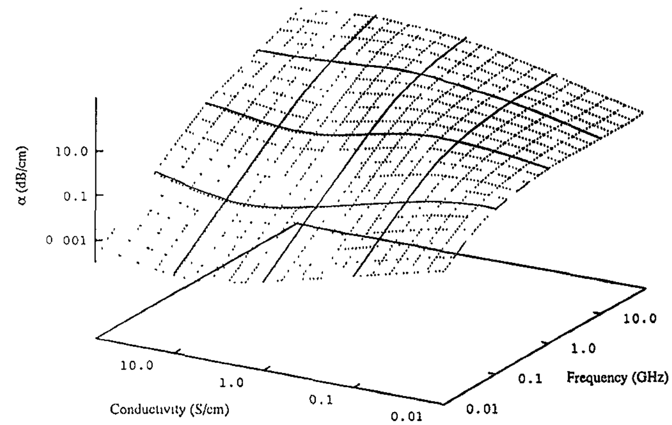

Fig. 2. Surface of attenuation constant a versus conductivity and frequency for microstrip geometry in [5]; dotted lines: new quasi-static model for $k = 3h + w/2$; solid lines: full-wave results.

The first example is the case considered originally by Hasegawa, which has frequently been used by others as a standard for comparison [5], [10]. The structure consists of a 160-μm wide microstrip, on a 1-μm thick silicon dioxide layer, on a 250-μm thick silicon substrate. Using the quasi-static model previously discussed, the surfaces of attenuation constant, α, and slow-wave factor, β/β_0, as a function of both f and σ, are shown in Figs. 2 and 3. Also shown in each figure are specific contours found using the full-wave calculations. For the attenuation constant, α (Fig. 2), the agreement between the quasi-static and full-wave calculations is excellent over the full four orders of magnitude of frequency and conductivity shown, covering all three domains of skin-effect, slow-wave, and dielectric quasi-TEM propagation. For the slow-wave factor, β/β_0, only at the very highest frequency and conductivity is there a noticeable difference (which is still less than 20%). In contrast, Mesa *et al.* [10], who used a more conventional quasi-static model that did not fully consider the impact of the semiconductor on Z, showed significant disagreement between full-wave and their quasi-static results, even at low frequency and conductivity. Our agreement has been achieved only by assuring that the quasi-statics account for changes in both Y and Z.

We have also verified the h/w and conductivity dependence of the model, keeping the frequency fixed at 1 GHz. For this example the linewidth is held constant at 50 μm, the thickness of the silicon dioxide layer is 1 μm, and the thickness of the silicon layer is varied from 10 μm to 1000 μm. For both the attenuation constant α and phase constant β, the quasi-static calculation was typically within 5% of the full-wave result, for a range of conductivity from 0.01 to 10 S/cm. Again, the agreement between the full-wave and quasi-static calculations is due to the use of (4) and (5) to find the surface impedance of the lossy semiconductor layer.

$$Z_{\text{semi}} = \frac{1}{jw}\sqrt{\frac{j\omega\mu_0}{j\omega\varepsilon_{\text{semi}} + \sigma} \frac{\boldsymbol{H}_0^{(2)}(j\beta_s b)\boldsymbol{H}_0^{(1)}(j\beta_s a) - \boldsymbol{H}_0^{(2)}(j\beta_s a)\boldsymbol{H}_0^{(1)}(j\beta_s b)}{\boldsymbol{H}_0^{(2)}(j\beta_s b)\boldsymbol{H}_1^{(1)}(j\beta_s a) - \boldsymbol{H}_1^{(2)}(j\beta_s a)\boldsymbol{H}_0^{(1)}(j\beta_s b)}}. \tag{4}$$

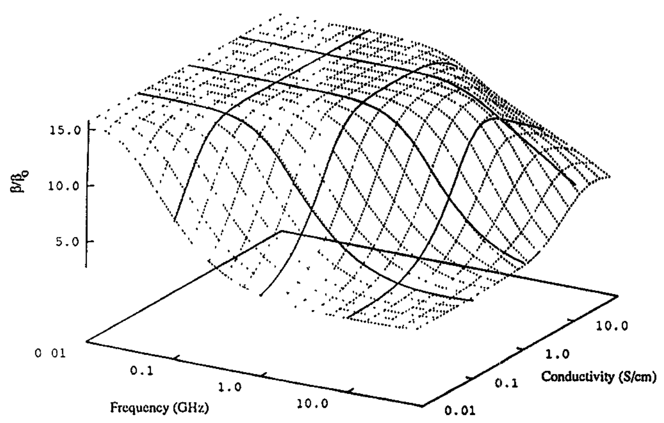

Fig. 3. Surface of slow-wave factor β/β_0 versus conductivity and frequency for microstrip geometry in [5]; dotted lines: new quasi-static model for $k = 3h + w/2$; solid lines: full-wave results.

IV. Conclusion

An accurate quasi-static model of a microstrip over a semiconductor layer has been developed. The model agrees with full-wave calculations in all three modes of propagation (skin-effect, slow-wave, and dielectric quasi-TEM), for both the attenuation constant α and the propagation constant β. The agreement between quasi-static and full-wave models suggests that even for the high-frequency, high-conductivity case, the behavior of the transmission line is still approximately quasi-TEM. Advantages of the quasi-static model include significant improvements in computational efficiency: on an IBM RISC System/6000 workstation, a single quasi-static-curve covering a frequency range of 0.01 to 100 GHz (at fixed conductivity and interconnect dimension) requires approximately 0.25 sec of CPU time; in contrast, the full-wave calculation required over 1000 sec of CPU time to obtain the same curve.

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